

STUDY OF 3-BUS ELECTRIC POWER SYSTEM POWER FLOW USING THE NEWTON-RAPHSON METHOD

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ABSTRACT

This study is a study of power flow in an electric power system consisting of three buses which intends to find the magnitude of voltage and angles of voltage on each bus, as well as the power loss on each line. Power system modeling for power flow studies produces mathematical equations that can be solved simultaneously. One of the numerical methods to solve this problem is to use the Newton-Raphson method.

To solve the power flow, the system is assumed to operate in a three-phase balanced condition and modeling is done for one phase which represents the other two phases. This requires that there are four quantities on each bus, namely active power and reactive power that enter the bus with these two quantities known on each bus. Furthermore, the magnitude of the voltage and the phase angle of the voltage at the bus, these two quantities will be calculated in the power flow study. The non-linear algebraic equation for the power flow model is linearized using the Taylor series to become a matrix equation in which there is a Jacobian matrix. This matrix equation is solved simultaneously (simultaneously) using the Newton-Raphson iteration method.

Iteration results show that the value of the voltage angle of Bus-2 is 1,33 electric-degre, the angle of voltage of Bus-3 is -5,38 electric-degre and the amplitude of the voltage of Bus-3 is 0.9343-pu. Bus-1 as Swing-Bus injects active power of 99.50 MW and reactive power of 26.00 MVar, Bus-2 as Bus-PV provides active power of 400.58 MW and reactive power of 418.57 MVar and Bus-3 as Bus-load (PQ-Bus) receives active power of 219.00 MW from Bus-1 and 281.08 MW to fulfill a scheduled active load of 50 0.08 MW and receives 143.14 MW of reactive power from Bus-1 and 206.97 MVar to meet the scheduled reactive power of 350.11 MVar. There is no active power loss on each line because in the calculation the line resistance is ignored, while reactive power losses are present on each line, namely line 1-2 of 7.27 MVar, line 1-3 of 31.36 MVar and line 2-3 of 55.83 MVar.

Keywords: active-power, reactive-power, bus-voltage, voltage-angle, power-line-loss.

INTRODUCTION

The electric power system is an electric power network consisting of a number of generating stations in a number of locations connected by a transmission network and/or distribution network to various loads (dimmand) that are spread geographically in various locations. In operation, the supply of electrical energy from power stations depends on load demand, which is dynamic in nature, which changes over time.

To find out how much electric power is required for each load bus, how much power is generated by each power station and also how much power is supplied by each channel/network, an analysis of the system is carried out, which is said to be a study or power flow analysis. This power flow study is carried

out with the assumption that it operates under balanced inter-phase load conditions and in normal operating conditions (steady-state), so that it is declared a single-phase system, (Saadat H., 1999; p.189). The benefits of this power flow study are to carry out development plans, operational steps (network maneuvers), economical generator scheduling and are also used for analysis of system transients and contingencies. The purpose of the power flow study is to obtain voltage values, voltage angles, active power and reactive power on each bus, as well as power losses on each channel, (Murty P.S.R., 2007; p. 98).

To get the value of each parameter in a power flow study, using electric circuit theory, namely Kirchoff's law and Ohm's law, with a large number of networks and buses (nodes) will produce non-linear equations that must be solved simultaneously (simultaneously). This equation is increasingly complex in proportion to the number of buses and channels that exist in the system. This computational problem is solved by numerical methods, one of which is the Newton-Raphson method. This method is often used for analysis of electric circuits in power flow studies because in the iteration process it quickly reaches convergence so that voltage values, voltage angles at each bus and others can be obtained with a low computational load on the computer. Iteration is a process by entering a certain initial value and completing it repeatedly until these values fulfill the equation.

LITERATURE REVIEW

Electrical Power System Modeling

An electric power system consisting of generating stations and load centers that are connected to one another, under normal operating conditions the load on each bus will change at any time. When one of the buses increases its load, system power will flow to that bus from one or more channels connected to it. Conversely, if the load decreases, the power flow to the bus decreases, or maybe the power flow flows to other buses whose load increases. As an illustration, review Figure 1.

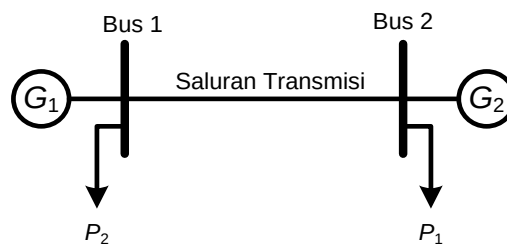


Figure 1 Schematic of the electric power system

If the load on P1 is less than or equal to the power of the generator G1 then the load on P1 is only borne by the generator G1, otherwise if the load on P1 is greater than the capacity of the generator G1 then to meet the needs of P1 the power is added from the generator G2 through the transmission line. This means that there is a flow of power from bus 2 to bus 1. In another condition, if the load P2 requires more power than generator G2, power will be added from generator G1 so that there is power flow from bus 1 to bus 2.

Power flow analysis is carried out when the system operates under normal conditions (steady-state) and the system is in a three-phase balanced condition so that it can be represented by a single-phase circuit. The results of the power flow study will obtain the voltage profile and angle at each bus, the amount of reactive power at certain buses, the power loss on the line and the amount of real (active)/reactive power injected into the system by the swing bus, (Del Toro, 1992).

In solving the problem of power flow, the system is assumed to operate in a three-phase balanced condition and modeling is done for one phase which represents the other two phases. The electric power system which is reviewed for power flow, the buses are categorized into three types of buses as follows:

1. PV bus, this type of bus has active power quantity (P) and voltage (V). This is the generating station bus to which a three-phase synchronous generator is connected. At this bus the power P is generated and the voltage V is determined. The results of the power flow study show that the reactive power Q supplied by the load to the synchronous generator is required to maintain the reference voltage magnitude V, and the reference voltage angle ϕ .
2. PQ bus, On this bus, the active power P and reactive power Q have been determined to be injected into the bus (PQ generation bus) or absorbed by the load connected to the bus. The power flow solution returns the magnitude of the bus voltage, V and the voltage angle, ϕ .
3. Swing bus, this bus is determined by the magnitude of the voltage and the voltage angle. The load flow solution returns the active power P and reactive power Q generated or absorbed on that bus to balance the load power, and the resulting losses. At least one bus in the model must be designated as a swing bus, but usually a single swing bus is required.

Power on each Bus

The active power and reactive power on bus i are:

$$P_i + jQ_i = V_i \cdot I_i^* \quad (1)$$

This is a form of power flow equation involving the variables active power, reactive power bus voltage and bus current. Then the current I_i is:

$$I_i = \frac{P_i - jQ_i}{V_i^*} \quad (2)$$

The power on bus i is caused by current I_i which is influenced by all admittances and bus voltages at the end of the line connected to bus i, then the current equation can be rewritten in the form of a bus admittance matrix, becoming:

$$I_i = \sum_{j=1}^n Y_{ij} \cdot V_j \quad (3)$$

and from equations (2) and (3), obtained:

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^n Y_{ij} \cdot V_j, \quad \text{for } (i = 1, \dots, n),$$

or

$$P_i - jQ_i = V_i^* \left(\sum_{j=1}^n Y_{ij} \cdot V_j \right), \quad (4)$$

This is a complex form of power at bus i, so that the real is the active power (P) and the imaginary part is the reactive power (Q) as follows,

$$P_i = \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j), \quad (5)$$

$$Q_i = - \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j), \quad (6)$$

Newton-Raphson Method (NR)

Equations (5) and (5) are a set of nonlinear algebraic equations for independent variables, the

magnitude of the voltage per unit, and the phase angle in radians. Linearization of these two equations using the Taylor series by ignoring all higher orders will result in a set of linear equations such as equation (7), (Aung Z. L., 2019)

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{(k)}}{\partial \delta_2} & \dots & \frac{\partial P_2^{(k)}}{\partial \delta_n} & \frac{\partial P_2^{(k)}}{\partial |V_2|} & \dots & \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n^{(k)}}{\partial \delta_2} & \dots & \frac{\partial P_n^{(k)}}{\partial \delta_n} & \frac{\partial P_n^{(k)}}{\partial |V_2|} & \dots & \frac{\partial P_n^{(k)}}{\partial |V_n|} \\ \frac{\partial Q_2^{(k)}}{\partial \delta_2} & \dots & \frac{\partial Q_2^{(k)}}{\partial \delta_n} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} & \dots & \frac{\partial Q_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n^{(k)}}{\partial \delta_2} & \dots & \frac{\partial Q_n^{(k)}}{\partial \delta_n} & \frac{\partial Q_n^{(k)}}{\partial |V_2|} & \dots & \frac{\partial Q_n^{(k)}}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}, \quad (7)$$

In the equation above, bus 1 is assumed to be a slack bus. The Jacobian matrix provides a linear relationship between the small change in the angle of stress $\Delta \delta_i^{(k)}$ and the magnitude of the stress $\Delta |V_i^{(k)}|$ with small changes in real and reactive power $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$. The elements of the Jacobian matrix are partial derivatives of equations (5) and (6), and evaluate to $\Delta \delta_i^{(k)}$ and $\Delta |V_i^{(k)}|$. In a short form, it can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}, \quad (8)$$

Diagonal elements and non-diagonal elements of J1 are calculated by the following equation:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq 1} |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j), \quad (9)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j), \quad j \neq i, \quad (10)$$

Diagonal elements and non-diagonal elements of J2 are calculated by the following equation:

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i| |Y_{ii}| \cos \theta_{ii} + \sum_{j \neq 1} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j), \quad (11)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j), \quad j \neq i, \quad (12)$$

Diagonal elements and non-diagonal elements of J3 are calculated by the following equation:

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq 1} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j), \quad (13)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j), \quad j \neq i, \quad (14)$$

Diagonal elements and non-diagonal elements of J4 are calculated by the following equation:

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i| |Y_{ii}| \sin \theta_{ii} - \sum_{j \neq 1} |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j), \quad (15)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j), \quad j \neq i, \quad (16)$$

The notation $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ is the difference between the scheduled value and the calculated

value and is known as the power difference, which is calculated by the equation:

$$\Delta P_i^{(k)} = P_i^{jdwl} - P_i^{(k)}, \quad (17)$$

$$\Delta Q_i^{(k)} = Q_i^{jdwl} - Q_i^{(k)}, \quad (18)$$

The new estimated value for the bus voltage is given in the following equation:

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta \delta_i^{(k)}, \quad (19)$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta |V_i^{(k)}|, \quad (20)$$

RESULTS AND DISCUSSION

3 Bus System Models

The power flow study uses the NR method which is calculated manually (not using an application program) taking the case of a three (3) bus power system. Bus 1 is a Swing-bus with known magnitude of the voltage at Bus-1 is $|V_1| = 1 + j0$ pu and the voltage angle is a reference for other buses so that $\delta_1 = 0^\circ$. Bus-2 is a generator bus with an active power of $P_2 = 400$ MW, the magnitude of the voltage at Bus-2 is $|V_2| = 1.03 + j0$ pu. Bus-3 is a load bus with an installed load $S_3 = P_3 + jQ_3 = 500$ MW + $j350$ MVar. The duct admittance is given in units per unit (pu) on a 100 MVA basis. Line resistance and charging line susceptance are neglected. The 3 bus power system diagram is shown in Figure 2.

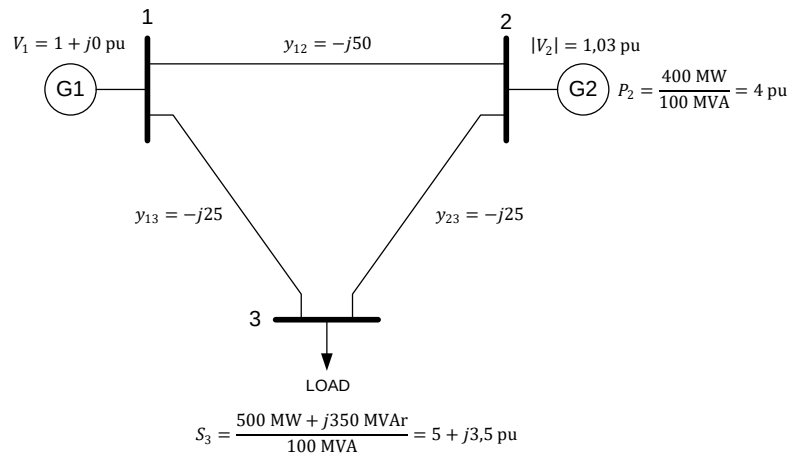


Figure 2 One-line diagram of a 3 bus power system

Known and unknown values in this system can be seen in Table 1.

Table 1 Value on each bus

BUS	KNOWLEDGE VALUE?	
	YES	NO
(1) Slack	V_1, δ_1	P_1, Q_1
(2) Generator	$P_2, V_2 $	Q_2, S_2
(3) Burden	P_3, Q_3	$ V_3 , \delta_3$

Bus admission:

$$Y_{11} = y_{12} + y_{13} = -j50 - j25 = -j75,$$

$$Y_{22} = y_{12} + y_{23} = -j50 - j25 = -j75,$$

$$Y_{33} = y_{13} + y_{23} = -j25 - j25 = -j50,$$

$$Y_{12} = Y_{21} = -y_{12} = -(-j50) = j50,$$

$$Y_{13} = Y_{31} = -y_{13} = -(-j25) = j25,$$

$$Y_{23} = Y_{32} = -y_{23} = -(-j25) = j25,$$

So that the bus admittance matrix can be expressed as follows,

$$Y_{BUS} = \begin{bmatrix} 75\angle-\frac{\pi}{2} & 50\angle\frac{\pi}{2} & 25\angle\frac{\pi}{2} \\ 50\angle\frac{\pi}{2} & 75\angle-\frac{\pi}{2} & 25\angle\frac{\pi}{2} \\ 25\angle\frac{\pi}{2} & 25\angle\frac{\pi}{2} & 50\angle-\frac{\pi}{2} \end{bmatrix}$$

The equation for real power and reactive power on each bus (1st to 3rd bus) is obtained as equations (5) and (6), obtained as follows,

$$\begin{aligned} P_2 &= |V_2| \cdot |V_1| \cdot |Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2 \cdot |Y_{22}| \cos(\theta_{22}) \\ &\quad + |V_2| \cdot |V_3| \cdot |Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3) \\ P_3 &= |V_3| \cdot |V_1| \cdot |Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3| \cdot |V_2| \cdot |Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) \\ &\quad + |V_3|^2 \cdot |Y_{33}| \cos(\theta_{33}) \\ Q_3 &= -\{|V_3| \cdot |V_1| \cdot |Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3| \cdot |V_2| \cdot |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) \\ &\quad + |V_3|^2 \cdot |Y_{33}| \sin(\theta_{33})\} \end{aligned}$$

Substitute values $|Y_{ij}|$ and θ_{ij} , so we get,

$$\begin{aligned} P_2 &= 50 \cdot |V_2| \cdot |V_1| \cos\left(\frac{\pi}{2} - \delta_2 + \delta_1\right) + 25 \cdot |V_2|^2 \cos\left(\frac{\pi}{2} - \delta_2 + \delta_3\right) \\ P_3 &= 25 \cdot |V_3| \cdot |V_1| \cos\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 25 \cdot |V_3| \cdot |V_2| \cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right) \\ Q_3 &= -\left\{25 \cdot |V_3| \cdot |V_1| \sin\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 25 \cdot |V_3| \cdot |V_2| \sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right) - 50 \cdot |V_3|^2\right\} \end{aligned}$$

Determination of the Jacobian Matrix

Jacobian matrix and its equations

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_3| \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}$$

The elements of the Jacobian matrix are obtained from the partial derivatives of P_2 , P_3 and Q_3 with respect to δ_2 , δ_3 and $|V_3|$,

$$\begin{aligned} \frac{\partial P_2}{\partial \delta_2} &= 50 \cdot |V_2| \cdot |V_1| \sin\left(\frac{\pi}{2} - \delta_2 + \delta_1\right) + 25 \cdot |V_2| \cdot |V_3| \sin\left(\frac{\pi}{2} - \delta_2 + \delta_3\right), \\ \frac{\partial P_2}{\partial \delta_3} &= -25 \cdot |V_2| \cdot |V_3| \sin\left(\frac{\pi}{2} - \delta_2 + \delta_3\right), \\ \frac{\partial P_2}{\partial |V_3|} &= 25 \cdot |V_2| \cos\left(\frac{\pi}{2} - \delta_2 + \delta_3\right), \\ \frac{\partial P_3}{\partial \delta_2} &= -25 \cdot |V_3| \cdot |V_2| \sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right), \\ \frac{\partial P_3}{\partial \delta_3} &= 25 \cdot |V_3| \cdot |V_1| \sin\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 25 \cdot |V_3| \cdot |V_2| \sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right), \\ \frac{\partial P_3}{\partial |V_3|} &= 25 \cdot |V_1| \cos\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 25 \cdot |V_2| \cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right), \end{aligned}$$

$$\begin{aligned}\frac{\partial Q_3}{\partial \delta_2} &= -25 \cdot |V_3| \cdot |V_2| \cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right), \\ \frac{\partial Q_3}{\partial \delta_3} &= 25 \cdot |V_3| \cdot |V_1| \cos\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 25 \cdot |V_3| \cdot |V_2| \cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right), \\ \frac{\partial Q_3}{\partial |V_3|} &= -25 \cdot |V_1| \sin\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) - 25 \cdot |V_2| \sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right) + 100 \cdot |V_3|,\end{aligned}$$

Scheduled power (schedule, sch) on each bus, namely the difference between the generator power and the load expressed in units per unit (pu), namely:

$$\begin{aligned}P_2^{sch} &= P_2^g - P_2^d = 4 - 0 = 4 \text{ pu}, \\ S_3^{sch} &= S_3^g - S_3^d = 0 - (5 + j3,5) = -5 - j3,5 \text{ pu},\end{aligned}$$

The slack-bus voltage is $|V_1| \angle \delta_1 = 1 \angle 0$ pu and the magnitude of the bus-2 voltage is $|V_2| = 1,03$ pu. For the iteration process starting with an initial value estimate,

- The magnitude of the voltage on the bus 3: $|V_3^{(0)}| = 1,0$,
- The voltage angle at the bus 3: $\delta_3^{(0)} = 0,0$, and
- The voltage angle at the bus 2: $\delta_2^{(0)} = 0,0$,

Look up the power on each bus with estimated values $k = 0$

$$\begin{aligned}P_2^{d(0)} &= 1.03 \cdot 1 \cdot 50 \cos\left(\frac{\pi}{2} - (0) + 0\right) + 1.03^2 \cdot 75 \cos\left(-\frac{\pi}{2}\right) + 1.03 \cdot (1) \cdot 25 \cos\left(\frac{\pi}{2} - (0) + (0)\right) = 0, \\ P_3^{d(0)} &= (1) \cdot 1 \cdot 25 \cos\left(\frac{\pi}{2} - (0) + 0\right) + (1) \cdot 1.03 \cdot 25 \cos\left(\frac{\pi}{2} - (0) + (0)\right) + (1)^2 \cdot 50 \cos\left(-\frac{\pi}{2}\right) = 0, \\ Q_3^{d(0)} &= -\left\{(1) \cdot 1 \cdot 25 \sin\left(\frac{\pi}{2} - (0) + 0\right) + (1) \cdot 1.03 \cdot 25 \sin\left(\frac{\pi}{2} - (0) + (0)\right) + (1)^2 \cdot 50 \sin\left(-\frac{\pi}{2}\right)\right\} = -0.75,\end{aligned}$$

The residual power is calculated based on the difference between the scheduled power (sch) and the power obtained from the calculated results by entering the initial estimated value $k = 0$

$$\begin{aligned}\Delta P_i^{(k)} &= P_i^{sch} - P_i^{(k)}, \\ \Delta Q_i^{(k)} &= Q_i^{sch} - Q_i^{(k)},\end{aligned}$$

obtained

$$\begin{aligned}\Delta P_2^{(0)} &= P_2^{sch} - P_2^{(0)} = 4 - 0 = 4, \\ \Delta P_3^{(0)} &= P_3^{sch} - P_3^{(0)} = (-5) - 0 = -5, \\ \Delta Q_3^{(0)} &= Q_3^{sch} - Q_3^{(0)} = (-3,5) - (-0,75) = -2,75,\end{aligned}$$

Look for the elements of the Jacobian matrix by entering the initial estimated value $k = 0$

$$\begin{aligned}\frac{\partial P_2^{(0)}}{\partial \delta_2} &= 50 \cdot 1,03 \cdot 1 \sin\left(\frac{\pi}{2} - (0) + 0\right) + 25 \cdot 1,03 \cdot (1) \sin\left(\frac{\pi}{2} - (0) + (0)\right) = \mathbf{77,25}, \\ \frac{\partial P_2^{(0)}}{\partial \delta_3} &= -25 \cdot 1,03 \cdot (1) \sin\left(\frac{\pi}{2} - (0) + (0)\right) = \mathbf{-25,75}, \\ \frac{\partial P_2^{(0)}}{\partial |V_3|} &= 25 \cdot 1,03 \cos\left(\frac{\pi}{2} - (0) + (0)\right) = \mathbf{0}, \\ \frac{\partial P_3^{(0)}}{\partial \delta_2} &= -25 \cdot (1) \cdot 1,03 \sin\left(\frac{\pi}{2} - (0) + (0)\right) = \mathbf{-25,75}, \\ \frac{\partial P_3^{(0)}}{\partial \delta_3} &= 25 \cdot (1) \cdot 1 \sin\left(\frac{\pi}{2} - (0) + (0)\right) + 25 \cdot (1) \cdot 1,03 \sin\left(\frac{\pi}{2} - 0 + (0)\right) = \mathbf{50,75}, \\ \frac{\partial P_3^{(0)}}{\partial |V_3|} &= 25 \cdot 1 \cos\left(\frac{\pi}{2} - (0) + 0\right) + 25 \cdot 1,03 \cos\left(\frac{\pi}{2} - (0) + (0)\right) = \mathbf{0}, \\ \frac{\partial Q_3^{(0)}}{\partial \delta_2} &= -25 \cdot (1) \cdot 1,03 \cos\left(\frac{\pi}{2} - (0) + (0)\right) = \mathbf{0},\end{aligned}$$

$$\begin{aligned}\frac{\partial Q_3^{(0)}}{\partial \delta_3} &= 25 \cdot (1) \cdot 1 \cos\left(\frac{\pi}{2} - (0) + (0)\right) + 25 \cdot (1) \cdot 1,03 \cos\left(\frac{\pi}{2} - (0) + (0)\right) = \mathbf{0}, \\ \frac{\partial Q_3^{(0)}}{\partial |V_3|} &= -25 \cdot 1 \sin\left(\frac{\pi}{2} - (0) + (0)\right) - 25 \cdot 1,03 \sin\left(\frac{\pi}{2} - (0) + (0)\right) + 100 \cdot (1) = \mathbf{49,25},\end{aligned}$$

After calculating the Jacobian matrix value with the initial estimated value $k = 0$, $\delta_2^{(0)} = 0$, $\delta_3^{(0)} = 0$, $|V_3^{(0)}| = 1$, are:

$$\begin{bmatrix} 4 \\ -5 \\ -2,75 \end{bmatrix} = \begin{bmatrix} 77,25 & -25,75 & 0 \\ -25,75 & 50,75 & 0 \\ 0 & 0 & 49,25 \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

By solving the matrix above we get:

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix} = \begin{bmatrix} 77,25 & -25,75 & 0 \\ -25,75 & 50,75 & 0 \\ 0 & 0 & 49,25 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 4 \\ -5 \\ -2,75 \end{bmatrix}$$

$$\begin{aligned}\Delta\delta_2^{(0)} &= 0,0228, \\ \Delta\delta_3^{(0)} &= -0,0870, \\ \Delta|V_3^{(0)}| &= -0,0558,\end{aligned}$$

The value of the phase angle of bus-2, the phase angle of bus-3 and the voltage of bus-3 for the estimation of $k=1$, are:

$$\begin{aligned}\delta_2^{(1)} &= \delta_2^{(0)} + \Delta\delta_2^{(0)} = 0 + 0,0228 = 0,0228 \text{ rad}, \\ \delta_3^{(1)} &= \delta_3^{(0)} + \Delta\delta_3^{(0)} = 0 + (-0,0870) = -0,0870 \text{ rad}, \\ |V_3^{(1)}| &= |V_3^{(0)}| + \Delta|V_3^{(0)}| = 1 + (-0,0558) = 0,9442 \text{ pu},\end{aligned}$$

Calculating the values of the elements of the Jacobian matrix such as $k=0$ iterations using the values $\delta_2^{(1)}$, $\delta_3^{(1)}$, $|V_3^{(1)}|$ obtained:

$$\begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_3^{(1)}| \end{bmatrix} = \begin{bmatrix} 77,65 & -24,17 & 2,82 \\ -24,17 & 47,68 & -4,99 \\ 2,66 & -4,72 & 43,92 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0,16 \\ -0,28 \\ -0,39 \end{bmatrix}$$

So obtained:

$$\begin{aligned}\Delta\delta_2^{(1)} &= 0,0003, \\ \Delta\delta_3^{(1)} &= -0,0067, \\ \Delta|V_3^{(1)}| &= -0,0096,\end{aligned}$$

The value of the phase angle of bus-2, the phase angle of bus-3 and the voltage of bus-3 for the estimated value of $k=2$, are:

$$\begin{aligned}\delta_2^{(2)} &= \delta_2^{(1)} + \Delta\delta_2^{(1)} = 0,0228 + 0,0003 = 0,0231 \text{ rad}, \\ \delta_3^{(2)} &= \delta_3^{(1)} + \Delta\delta_3^{(1)} = -0,0870 + (-0,0067) = -0,0937 \text{ rad}, \\ |V_3^{(2)}| &= |V_3^{(1)}| + \Delta|V_3^{(1)}| = 0,9442 + (-0,0096) = 0,9346 \text{ pu},\end{aligned}$$

Calculates the values of the elements of the Jacobian matrix using the values $\delta_2^{(2)}$, $\delta_3^{(2)}$, $|V_3^{(2)}|$ got:

$$\begin{bmatrix} \Delta\delta_2^{(2)} \\ \Delta\delta_3^{(2)} \\ \Delta|V_3^{(2)}| \end{bmatrix} = \begin{bmatrix} 75,38 & -23,90 & 3,00 \\ -23,90 & 47,16 & -5,34 \\ 2,80 & -4,99 & 42,99 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0,01 \\ -0,01 \\ -0,01 \end{bmatrix}$$

obtained:

$$\begin{aligned}\Delta\delta_2^{(2)} &= 0,0792 \cdot 10^3, \\ \Delta\delta_3^{(2)} &= -0,2015 \cdot 10^3, \\ \Delta|V_3^{(2)}| &= -0,2612 \cdot 10^3,\end{aligned}$$

The value of the phase angle of bus-2, phase angle of bus-3 and voltage of bus-3 for the 3rd iteration with the value of the 2nd iteration is:

$$\begin{aligned}\delta_2^{(3)} &= \delta_2^{(2)} + \Delta\delta_2^{(2)} = 0,0231 + 0,0792 \cdot 10^3 = \mathbf{0,0232 \text{ rad} = 1,33^\circ}, \\ \delta_3^{(3)} &= \delta_3^{(2)} + \Delta\delta_3^{(2)} = -0,0937 + (-0,2015 \cdot 10^3) = \mathbf{-0,0939 \text{ rad} = -5,38^\circ}, \\ |V_3^{(3)}| &= |V_3^{(2)}| + \Delta|V_3^{(2)}| = 0,9346 + (-0,2612 \cdot 10^3) = \mathbf{0,9343 \text{ pu}},\end{aligned}$$

These values are the result of iterations using the Newton-Raphson method which has converged so that they can be used to calculate the active power and reactive power at each bus.

$$\begin{aligned}P_2 &= 50 \cdot 1,03 \cdot 1 \cos\left(\frac{\pi}{2} - (0,0232) + 0\right) + 25 \cdot 1,03 \cdot (0,9343) \cos\left(\frac{\pi}{2} - (0,0232) + (-0,0939)\right), \\ &= \mathbf{4,0055 \text{ pu}}, \\ P_3 &= 25 \cdot (0,9343) \cdot 1 \cos\left(\frac{\pi}{2} - (-0,0939) + 0\right) + 25 \cdot (0,9343) \cdot 1,03 \cos\left(\frac{\pi}{2} - (-0,0939) + (0,0232)\right), \\ &= \mathbf{-5,0008 \text{ pu}}, \\ Q_3 &= -\left\{25 \cdot (0,9343) \cdot 1 \sin\left(\frac{\pi}{2} - (-0,0939) + 0\right) \right. \\ &\quad \left. + 25 \cdot (0,9343) \cdot 1,03 \sin\left(\frac{\pi}{2} - (-0,0939) + (0,0232)\right) - 50 \cdot (0,9343)^2\right\} = \mathbf{-3,5022 \text{ pu}},\end{aligned}$$

Calculating Power on Each Bus

Power in MW and MVar:

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j \text{ untuk } j \neq i$$

$$P_i - jQ_i = V_i^* \cdot [V_i \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j],$$

Calculating power on the bus:

$$\begin{aligned}V_i &= |V_i| \angle \delta_i, \\ V_i &= |V_i| \cdot (\cos \delta_i + j \sin \delta_i), \\ V_1 &= |V_1| \angle \delta_1 = 1 \angle 0, \\ V_1 &= |V_1| \cdot (\cos \delta_1 + j \sin \delta_1) = 1 \cdot (\cos 0 + j \sin 0) = 1 + j0, \\ V_2 &= |V_2| \angle \delta_2 = 1,03 \angle 0,0232, \\ V_2 &= |V_2| \cdot (\cos \delta_2 + j \sin \delta_2) = 1,03 \cdot (\cos 0,0232 + j \sin 0,0232) = 1,0297 + j0,0239, \\ V_3 &= |V_3| \angle \delta_3 = 0,9343 \angle (-0,0939), \\ V_3 &= |V_3| \cdot (\cos \delta_3 + j \sin \delta_3) = 0,9343 \cdot (\cos(-0,0939) + j \sin(-0,0939)) = 0,9302 - j0,0876,\end{aligned}$$

Power on bus 1:

$$\begin{aligned}P_1 - jQ_1 &= V_1^* \cdot [V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \cdot \text{MVA}_{base}, \\ &= (1 - j0) \cdot [(1 + j0)((0 - j50) + (0 - j25)) \\ &\quad - ((0 - j50)(1,0297 + j0,0239) + (0 - j25)(0,9302 - j0,0876))] \cdot 100, \\ &= \mathbf{99,50 \text{ MW} - j26 \text{ MVar}},\end{aligned}$$

Power on bus 2:

$$\begin{aligned}P_2 - jQ_2 &= V_2^* \cdot [V_2(y_{21} + y_{23}) - (y_{21}V_1 + y_{23}V_3)] \cdot \text{MVA}_{base} \\ &= (1,0297 - j0,0239) \cdot [(1,0297 + j0,0239)((0 - j50) + (0 - j25)) \\ &\quad - ((0 - j50)(1 + j0) + (0 - j25)(0,9302 - j0,0876))] \cdot 100, \\ &= \mathbf{400,58 \text{ MW} - j418,57 \text{ MVar}},\end{aligned}$$

Power on bus 3:

$$\begin{aligned} P_3 - jQ_3 &= V_3^* \cdot [V_3(y_{31} + y_{32}) - (y_{31}V_1 + y_{32}V_2)] \cdot \text{MVA}_{base}, \\ &= (0.9302 + j0.0876) \cdot [(0.9302 - j0.0876)((0 - j25) + (0 - j25)) \\ &\quad - ((0 - j25)(1 + j0) + (0 - j25)(1.0297 + j0.0239))] \cdot 100, \\ &= -500,08 \text{ MW} + j350,10 \text{ MVar} \end{aligned}$$

Calculate line current

$$\begin{aligned} I_{ij} &= y_{ij} \cdot (V_i - V_j), \\ I_{ji} &= -I_{ij}, \end{aligned}$$

Channel current 1 – 2:

$$\begin{aligned} I_{12} &= y_{12} \cdot (V_1 - V_2) = (0 - j50) \cdot [(1 + j0) - (1,0297 + j0,0239)] = -1,1950 + j1,4850 \text{ pu}, \\ I_{21} &= -I_{12} = 1,1950 - j1,4850 \text{ pu}, \end{aligned}$$

Channel current 1 – 3:

$$\begin{aligned} I_{13} &= y_{13} \cdot (V_1 - V_3) = (0 - j25) \cdot [(1 + j0) - (0,9302 - j0,0876)] = 2,1900 - j1,7450 \text{ pu}, \\ I_{31} &= -I_{13} = -2,1900 + j1,7450 \text{ pu}, \end{aligned}$$

Channel current 2 – 3:

$$\begin{aligned} I_{23} &= y_{23} \cdot (V_2 - V_3) = (0 - j25) \cdot [(1,0297 + j0,0239) - (0,9302 - j0,0876)] = 2,7875 - j2,4875 \text{ pu}, \\ I_{32} &= -I_{23} = -2,7875 + j2,4875 \text{ pu}, \end{aligned}$$

Calculate power flow between buses and line power loss

Power Flow Between Buses

Bus 1 – 2:

$$\begin{aligned} S_{12} &= V_1 \cdot I_{12}^* = (1 + j0) \cdot (-1,1950 - j1,4850) \cdot 100 = -119,50 \text{ MW} - j148,50 \text{ MVar}, \\ S_{21} &= V_2 \cdot I_{21}^* = (1,0297 + j0,0239) \cdot (1,1950 + j1,4850) \cdot 100 = 119,50 \text{ MW} + j155,77 \text{ MVar}, \end{aligned}$$

Bus 1 – 3:

$$\begin{aligned} S_{13} &= V_1 \cdot I_{13}^* = (1 + j0) \cdot (2,1900 + j1,7450) \cdot 100 = 219,00 \text{ MW} + j174,50 \text{ MVar}, \\ S_{31} &= V_3 \cdot I_{31}^* = (0,9302 - j0,0876) \cdot (-2,1900 - j1,7450) \cdot 100 = -219,00 \text{ MW} - j143,14 \text{ MVar}, \end{aligned}$$

Bus 2 – 3:

$$\begin{aligned} S_{23} &= V_2 \cdot I_{23}^* = (1,0297 + j0,0239) \cdot (2,7875 + j2,4875) \cdot 100 = 281,08 \text{ MW} + j262,80 \text{ MVar}, \\ S_{32} &= V_3 \cdot I_{32}^* = (0,9302 - j0,0876) \cdot (-2,7875 - j2,4875) \cdot 100 = -281,08 \text{ MW} - j206,97 \text{ MVar}, \end{aligned}$$

Power loss in the line

$$\begin{aligned} SL_{12} &= S_{12} + S_{21} = (-119,50 \text{ MW} - j148,50 \text{ MVar}) + (119,50 \text{ MW} + j155,77 \text{ MVar}), \\ &= 0,0000 \text{ MW} + j7,2665 \text{ MVar} \\ SL_{13} &= S_{13} + S_{31} = (219,00 \text{ MW} + j174,50 \text{ MVar}) + (-219,00 \text{ MW} - j143,14 \text{ MVar}), \\ &= 0,0000 \text{ MW} + j31,3645 \text{ MVar}, \\ SL_{23} &= S_{23} + S_{32} = (281,08 \text{ MW} + j262,80 \text{ MVar}) + (-281,08 \text{ MW} - j206,97 \text{ MVar}), \\ &= 0,0000 \text{ MW} + j55,8312 \text{ MVar} \end{aligned}$$

The results of calculating the voltage angle at Bus-2 and the voltage and voltage angle at Bus-3 through iterations with the NR method, the results can be found for the power at each bus. On Bus-1 the generator provides an active power of 99.50 MW and a reactive power of 26.00 MVar, the incoming power from Bus-2 to Bus-1 is 119.50 MW and 148.50 MVar, then Bus-1 provides 219, 00 MW and 174.50 MVar to Bus-3. Generator Bus-2 provides active power of 400.58 MW and reactive power of 418.57 MVar, 119.50 MW and 155.77 MVar are supplied to Bus-1 and 281.08 MW and 262.80 MVar are supplied to Bus-3. Bus-3 as a load bus receives 219.00 MW and 143.14 MVar of power from Bus-

1 and 281.08 MW and 206.97 MVar of Bus-2.

Active power loss in each channel is zero, only reactive power loss, namely channels 1-2 worth 7.27 MVar, channels 1-3 worth 31.36 MVar and channels 2-3 worth 55.83 MVar. The power flow diagram of the results of the analysis using the Newton-Raphson method is shown in Figure 3.

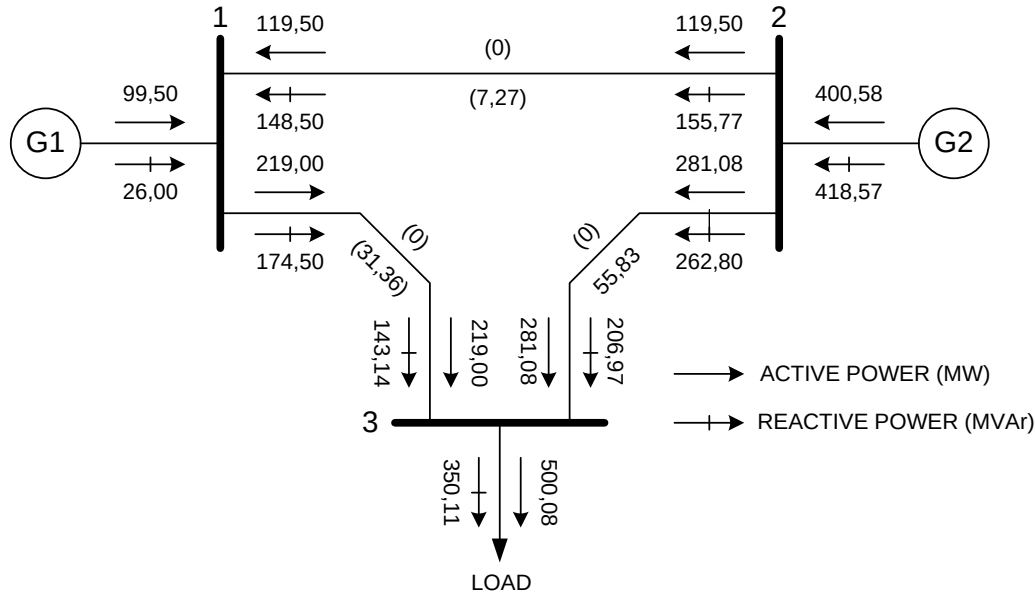


Figure 3 Power flow diagram of a 3 bus electric power system

CONCLUSION

From the results of the discussion for the 3 bus electric power system it can be concluded that:

1. The iteration is carried out three times and has reached convergence at the value of the Bus-2 voltage angle is $\delta_2^{(3)} = 0,0232 \text{ rad} = 1.33^\circ$ the Bus-3 voltage angle is $\delta_3^{(3)} = -0,0939 \text{ rad} = -5,38^\circ$ and the Bus-3 voltage amplitude is $|V_3^{(3)}| = 0,9343 \text{ pu}$
2. Bus-1 as Swing-Bus injects active power of 99.50 MW and reactive power of 26.00 MVar, Bus-2 as Bus-PV injects active power of 400.58 MW and reactive power of 418.57 MVar and Bus-3 as Bus- load (PQ-Bus) receives active power of 219.00 MW from Bus-1 and 281.08 MW to meet the scheduled active load of 500.08 MW and receives reactive power of 143.14 MW from Bus-1 and 206.97 MVar for meet the scheduled reactive power of 350.11 MVar.
3. There is no active power loss on each channel because in the calculation the channel resistance is ignored, while the reactive power loss is present on each channel, namely channels 1-2 of 7.27 MVar, channels 1-3 of 31.36 MVar and channels 2-3 of 55. 83 MVar.

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